**Phys 410**
**Fall 2013**
**Lecture #6 Summary**
**19 September, 2013**

Energy comes in many forms. We first encounter kinetic energy $T = \frac{1}{2}mv^2$. The kinetic energy of a single particle can change when it is acted upon by a force that has a component along the direction of displacement of the particle: $dT = \vec{F} \cdot d\vec{r}$. . This leads to the Work-Kinetic energy theorem: $T_2 - T_1 = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(r') \cdot d\vec{r}'$, where the value of the line integral (known as 'work') will in general depend on the path taken between the points $\vec{r}_1$ and $\vec{r}_2$.

There are two types of forces – conservative and non-conservative. Conservative forces have potential energy functions associated with them. To be conservative, a force must satisfy two requirements:

1) The force depends only on the particle coordinates, and not the velocity, momentum, time, etc.
2) The work done by the force between any two points must be independent of path.

Examples of conservative forces include gravity and the electrostatic force. Non-conservative forces include friction and the drag forces that we considered earlier.

The potential energy is defined as follows. Choose an arbitrary position $\vec{r}_0$ where the potential energy will be assigned a value of 0. The potential energy is defined in terms of the work done on the particle to take it from $\vec{r}_0$ to any point $\vec{r}$: $U(\vec{r}) \equiv -W(\vec{r}_0 \to \vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(r') \cdot d\vec{r}'$ , where there is no need to specify the contour for the line integral. Note the minus sign. With this definition, one can show that if only conservative forces act, the total mechanical energy $E = T + U$ of the system is conserved, i.e. $\Delta E = 0$. This conservation law is very useful for solving problems. If non-conservative forces do act, along with conservative forces, then the mechanical energy of the system changes by an amount equal to the work done by the non-conservative forces: $\Delta E = W_{nc}$. $W_{nc}$ is typically negative because non-conservative forces usually 'steal' mechanical energy and covert it to heat (thermal energy).

We considered the process of deducing a vector force from a given scalar potential-energy function. This is done through the gradient differential operator $\vec{F} = -\vec{\nabla}U$. Note that this is actually three equations in one. We did the example of the electrostatic potential $U = kq_1q_2/r$, and showed that the associated force is the Coulomb electrostatic force $\vec{F} = kq_1q_1\hat{r}/r^2$. Since conservative forces are derived from a potential energy function, we can find a simple necessary (but not sufficient) test to see if the force really is conservative. Taking the curl of a conservative force yields $\vec{\nabla} \times \vec{F} = -\vec{\nabla} \times \vec{\nabla} U = 0$, where the last term is a vector identity good for all scalar functions $U(\vec{r})$. Hence all conservative forces must be curl-free vector fields. An additional requirement is that the force depends only on the particle coordinates.

We considered energy for motion in one-dimensional systems. This is not as artificial as it first appears – later we will be able to break certain 3D problems to simpler 2D and 1D problems, and the methods that follow will be very useful. Consider a particle confined to move only on the x-axis. It has a kinetic energy $T = \frac{1}{2} m\dot{x}^2$. The kinetic energy can be altered by applying a force and doing work on the particle. The tangential component of the force does work as $W(x_0 \to x_1) = \int_{x_0}^{x_1} F_{\tan}(x')\,dx'$. If this force is conservative, one can define an associated potential energy (PE) as $U(x) = -W(x_0 \to x) = -\int_{x_0}^{x} F_{\tan}(x')\,dx'$, where it is assumed that $U(x_0) = 0$. We also expect that the total mechanical energy will be conserved: $E = T + U(x)$, and $\Delta E = 0$. This conservation law allows elegant solution of 1D problems involving conservative forces.

We did an example of a Hooke's law restoring force in 1D: $\vec{F} = -k\,x\,\hat{x}$., with an equilibrium point at $x = 0$. The corresponding potential is $U(x) = \frac{1}{2} kx^2$, with $U(0) = 0$. The mechanical energy is conserved: $E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2$. As the particle moves it exchanges energy back and forth between kinetic and potential energies.

Note that the force can be derived from the potential energy function through the 1D gradient, which is a total derivative: $F_{\tan} = -dU(x)/dx$.

The energy landscape created by the function $U(x)$ is very revealing. If there is a maximum or minimum in $U(x)$ it means that the driving force at that location is $F_{\tan} = 0$. As such, this represents an equilibrium point. A minimum in $U(x)$ is a stable equilibrium because a small displacement will result in forces that point back to the equilibrium point. This is the case when $d^2U(x)/dx^2 > 0$. A local maximum in $U(x)$ is unstable because a small displacement in either direction produces forces that draw the particle further away. This is the case with $d^2U(x)/dx^2 < 0$.